 Seat No. ________

B. Sc. (CBCS) (Sem. V) Examination

October / November – 2016

Statistics : S-502

(Mathematical Statistics)

[New Course]

Faculty Code : 003

Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) Q. No. 1 carries 20 marks, Q. No. 2 and Q. No. 3 each carries 25 marks.

(ii) Students can use their own scientific calculator.

1 Fill in the blanks and short questions : (Each 1 mark) **20**

(1) _____ is a characteristic function of Poisson distribution.

(2) _____ is a characteristic function of standard normal distribution.

(3) _____ is a characteristic function of geometric distribution.

(4) _____ is a moment generating function of normal distribution.

(5) _____ is a moment generating function of $\gamma(\alpha, p)$.

(6) _____ is a moment generating function of chi-square distribution.

(7) For normal distribution $\mu_{2n} =$ _______.

(8) For normal distribution $\mu_4 = k_4 + 3k_2^2$ is _______.

(9) If two independent variates $X_1 \sim N\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim N\left(\mu_2, \sigma_2^2\right)$ then $X_1 - X_2$ is distributed as ______.

(10) If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1 + X_2}$ is distributed as _______.

(11) If two independent variates $X_1 \sim \Lambda\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim \Lambda\left(\mu_2, \sigma_2^2\right)$ then $X_1 \cdot X_2$ is distributed as _______.

(12) Weibull distribution has application in _______.

(13) If χ_1^2 and χ_2^2 are two independent Chi-sqaure variates with d.f. n_1 and n_2, respectively, then the distribution of $\frac{\chi_1^2}{\chi_2^2}$ is _______.

(14) The range of multiple correlation coefficient R is ______.

(15) The range of partial regression coefficient is ______.

(16) Define Caush's distribution.

(17) Define Log normal with $\log_e(x-a)$ distribution.

(18) Write mean and variance of Gamma distribution with parameter (α, p).

(19) Write mean and variance of Weibul distribution.

(20) Write mean and variance of Lapace (double) exponential distribution.

2 (a) Write the answers of any three (Each 2 marks) **6**

(1) Define convergence in probability.

(2) If $u = \frac{x-a}{h}$, a and h being constants then

$$\varnothing_u(t) = e^{\left(-iat/h\right)} \varnothing_x\left(t/h\right).$$

(3) Define Weibul distribution.

(4) Usual notation of multiple correlation and multiple regression, prove that $\sum X_{1.23} x_2 = 0$.

(5) Prove that $b_{12.3} = \frac{b_{12} - b_{13}b_{23}}{1 - b_{13}b_{23}}$.

(6) In trivariate distribution it is found that $r_{12} = 0.86, r_{13} = 0.65$ and $r_{23} = 0.72$.

Find : (i) $r_{12.3}$ (ii) $R_{1.23}$.

(b) Write the answers of any three : (Each 3 marks) **9**

(1) Prove that $\mu_r = (-i)^r \left[\frac{d^r}{dt^r} \varnothing_u(t) \right]_{t=0}$; where $u = x - \mu$.

(2) Obtain probability density function for the characteristic function $\varnothing_x(t) = p\left(1 - qe^{it}\right)^{-1}$.

(3) Define exponential distribution and obtain its MGF. From MGF obtain its mean and variance.

(4) Define truncated Poisson distribution and also obtain its mean and variance.

(5) With usual notation of multiple correlation and multiple regression, prove that $b_{12} = \frac{b_{12.3} + b_{13.2}\, b_{32.1}}{1 - b_{13.2}\, b_{31.2}}$.

(6) With usual notation of multiple correlation and multiple regression, prove that $\sigma_{1.23}^2 = \sigma_1^2 \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$.

(c) Write the answers of any two : (Each 5 marks) **10**

(1) State and prove Chebchev's inequality.

(2) Derive t-distribution.

(3) Derive χ^2 distribution and show that $2\beta_2 - \beta_1 - 6 = 0$.

(4) Obtain marginal distribution of x for Bi-variate distribution.

(5) With usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{23}\, r_{13}}{1 - r_{23}^2}$.

3 (a) Write the answers of any three : (Each 2 marks) **6**

(1) Define Beta-I and Beta-II distribution.

(2) Obtain characteristic function of Poisson distribution with parameter λ.

(3) Define truncated distribution.

(4) With usual notation of multiple correlation and multiple regression, prove that $\sum X_{1.2}\, X_{3.12} = 0$.

(5) Prove that $\sigma_{3.12}^2 = \frac{\sigma_3^2\left(1-r_{12}^2-r_{23}^2-r_{13}^2+2r_{12}r_{23}r_{13}\right)}{\left(1-r_{12}^2\right)}$.

(6) In trivariate distribution it is found that

$\sigma_1 = 2, \sigma_2 = \sigma_3 = 3, r_{12} = 0.7, r_{23} = r_{31} = 0.5$.

Find : (i) $b_{13.2}$ (ii) $\sigma_{3.12}$.

(b) Write the answers of any three : (Each 3 marks) **9**

(1) Prove that $\mu_r' = (-i)^r \left[\frac{d^r}{dt^r}\varnothing_X(t)\right]_{t=0}$.

(2) Obtain MGF of normal distribution.

(3) Obtain mean and variance of uniform distribution.

(4) Define truncated binomial distribution and also obtain its mean and variance.

(5) With usual notations of multiple correlation and multiple regression, prove that

If $r_{12} = r_{23} = r_{31} = r$ then

$R_{1.23} = R_{2.31} = R_{3.12} = \frac{\sqrt{2}r}{\sqrt{1+r}}$.

(6) With usual notations of multiple correlation and multiple regression, prove that

$b_{12.3}\, b_{23.1}\, b_{31.2} = r_{12.3}\, r_{23.1}\, r_{31.2}$.

(c) Write the answers of any two : (Each 5 marks) **10**

(1) Obtain MGF of Gamma distribution with parameters α and p. Also show that $3\beta_1 - 2\beta_2 + 6 = 0$.

(2) Derive normal distribution.

(3) Derive F-distribution.

(4) Obtain conditional distribution of y when x is given for Bi-variate distribution.

(5) With usual notations of multiple correlation and multiple regression, prove that $r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}}$.